1. Form the differential equation by eliminating arbitrary constants $a$, $b$ from $y = ae^{3x} + be^{5x}$.

2. Solve $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

3. Solve $y'' - y = 0$, when $y = 0$ and $y' = 2$ at $x = 0$.

4. Find the particular integral of $(D^2 + 1)y = 8e^{-x}$.

5. Classify the singular points of $(1 - x^2)y'' - 2xy' + 2y = 0$.

6. Prove that $P_n(1) = 1$.

7. Show that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.

8. Prove that $\int_{0}^{\infty} x^C C^s dx = \frac{(C + 1)}{(\log C)^{C+1}}, C > 1$.

9. Find the Laplace transform of $e^t \cos t$.

10. Find inverse Laplace transform of $\frac{s^2 - s + 2}{s(s-3)(s+2)}$.

11. a) Find the orthogonal trajectories of $r = ce^\theta$, where $C$ is the parameter.  
    b) Solve $\frac{dy}{dx} - y = y^2 (\sin x + \cos x)$.

12. a) Using the method of variation of parameters solve $(D^2 + 1)y = x$.  
    b) Solve $(D^2 - 4D + 2)y = 12e^x \sin 2x$.

13. Obtain the series solution of the equation $x^2 y'' + xy' + (x^2 - 4)y = 0$ about $x = 0$.

14. a) Prove that $\beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma(m + n)}$  
    b) Prove that $\int J_0(x) J_1(x) dx = -\frac{1}{2} J_0^2(x)$.
15 a) Apply convolution theorem to evaluate

\[
L^{-1} \left[ \frac{1}{(s^2 + 1)(s^2 + 4)} \right].
\]

b) Use Laplace transform to solve \( y' - y = e^x \) given that \( y(0) = 1 \).

16 a) Find the general solution and singular solution of the Clairaut’s equation
\[ y = (x - a)p - p^2. \]
b) Solve the initial value problem \( y'' - 2y' + 3y = 0 \) with \( y(0) = 1, \ y'(0) = 0 \).

17 a) Prove that
\[
\int_{-1}^{1} P_n(x)P_m(x)dx = 0 \quad \text{if} \quad m \neq n.
\]
b) Find the Laplace transform of \( t \sin^2(3t) \).

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