PART – A (25 Marks)

1. Find the Taylor’s series expansion of \( f(x) = 2^x \) about \( x=0 \). (2)

2. Find the radius of curvature of the curve \( r = a \sin \theta + b \cos \theta \) at \( \theta = \pi/2 \). (3)

3. Show that \( \lim_{(x,y) \to (0,0)} \frac{x^2 - xy}{x^2 + y} \) does not exist. (2)

4. If \( z = y + f(u) \), \( u = \frac{x}{y} \), show that \( \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y \partial x} = 1 \). (3)

5. Evaluate \( \int_0^2 \int_{2y}^1 e^x \, dx \, dy \) by changing the order of integration. (2)

6. Find a vector that gives the direction of maximum rate of increase for \( f(x,y,z) = 6xyz \) at \((-1,2,1)\). (3)

7. Find the values of \( \lambda \) and \( \mu \) such that the system of equations \( x+y+z = 6, x+2y+3z = 10, x + 2y + \lambda z = \mu \) has an infinite number of solutions. (2)

8. Show that the vectors \((2,2,0), (3,0,2), (2,-2,2)\) are linearly independent. (3)

9. Discuss the convergence of the series \( \sum_{n=1}^{\infty} \left(1+\frac{1}{n^p}\right)^n \), \( p > 0 \). (2)

10. Test whether the series \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n \sqrt{n}} \) converges absolutely or not. (3)

PART – B (50 Marks)

11. (a) State and prove Rolle’s theorem. (6)

(b) Find the envelope of the family of curves \( x \tan \alpha + y \sec \alpha = 5 \), \( \alpha \) is a parameter. (4)

12. (a) Trace the curve \( y = x^3 - 12x - 16 \). (6)

(b) Examine \( f(x,y) = x^4 + 2x^2y - x^2 + 3y^2 \) for maximum and minimum values. (4)

13. (a) Show that \( \nabla = 12xi - 15y^2 j + k \) is irrotational and find a scalar function \( f(x,y,z) \) such that \( \nabla = \text{grad} \, f \). (5)

(b) Use the divergence theorem to evaluate \( \int_S \int \overline{F} \cdot \mathbf{n} \, ds \), where \( \overline{F} = 4xi - 2y^2j + z^2k \) and \( S \) is the surface bounding the region \( x^2+y^2 = 4, z=0 \) and \( z=3 \). (5)
14.(a) If -4, 10, $\sqrt{2}$ are the three eigen values of $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 2 & 1 \\ 4 & 3 & 1 & 2 \end{bmatrix}$, find the eigen values of $A^{-1}$.

(b) Find the canonical form, nature, index and signature of the quadratic form 

$$Q = 8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 - 8x_2x_3 + 4x_3x_1.$$ 

15. Test the convergence of the series

a) $\frac{1}{1.3.5} + \frac{2}{3.5.7} + \frac{3}{5.7.9} + ...$

b) $\sum \frac{(n!)^2}{(2n)!} x^{2n}$

16.(a) Find the evolute of the curve $y^2 = 4ax$.

(b) For the function $f(x,y) = \begin{cases} \frac{x^2y(x-y)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0), \end{cases}$

show that $\frac{\partial^2 f}{\partial x \partial y} \neq \frac{\partial^2 f}{\partial y \partial x}$ at $(0,0)$.

17.(a) Show that $\nabla x(\nabla \cdot \nabla) = \nabla \left( \nabla \cdot \nabla \right) - \nabla \cdot \nabla^2 \cdot \nabla$.

(b) Find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & 1 & 0 & 4 \\ 3 & 1 & 2 & -1 & 1 \\ 4 & -1 & 3 & -2 & -2 \\ 5 & 4 & 3 & -1 & 5 \end{bmatrix}$. 

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