FACULTY OF ENGINEERING & INFORMATICS
B.E.(Common to all Branches) I-Year (Supplementary) Examination, December 2013

Subject: Mathematics – II

Note: Answer all questions from Part-A. Answer any FIVE questions from Part-B.

PART – A (25 Marks)

1. Find an integrating factor of \( \left( x^2 - 2xy^2 \right) dx - \left( x^3 - 3x^2 y \right) dy = 0 \).

2. Solve \( \left( \frac{e^{-\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} \right) \frac{dx}{dy} = 1 \).

3. Particular integral of \( \frac{dy}{dx} + y = \cosh 3x \) is ________

4. Solve \( \left( D^2 + 4D + 5 \right) y = 2e^{-2x} \).

5. Express \( 1 + x - x^2 \) in terms of Legendre’s polynomials \( P_n(x) \).

6. Show that \( J_{1/2}(x) = J_{-1/2}(x) \tan x \).

7. Show that \( U_{n+1}(x) = 2xU_n(x) - U_{n-1}(x) \).

8. Prove that \( \sin(n\pi) = \frac{\pi}{\sin(n\pi)} \).

9. Find the Laplace transform of \( t^2 e^{-3t} \).

10. Apply convolution theorem to evaluate \( L^{-1} \left\{ \frac{1}{s(s+1)} \right\} \).

PART – B (50 Marks)

11.a) Solve \( \frac{dy}{dx} = e^{-y} \left( e^x - e^y \right) \).

b) Find the orthogonal trajectories of the family of curves \( \frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1 \), \( \lambda \) being a parameter.

12.a) Solve \( y^2 + 4y' + 4y = 3\sin x + 4\cos x \), \( y(0) = 1 \) and \( y'(0) = 0 \).

b) Solve, by the method of variation of parameters.

13. Obtain the series solution of the equation.
\[ x(1-x) \frac{d^2y}{dx^2} - (1+3x) \frac{dy}{dx} - y = 0 \] about \( x = 0 \)

14.a) Prove \( \frac{1-z^2}{(1-2xz+z^2)^{3/2}} = \sum_{n=0}^{\infty} (2n+1)z^n P_n(x) \).

b) Show that \( \int_{-1}^{1} P_n(x) \ dx = \frac{2}{2n+1} \).

15.a) State and prove the orthogonal property of Chebyshev polynomials of the second kind.

b) Express \( x^3 + 3x^2 - 2x + 1 \) in terms of Chebyshev polynomials of the first kind.

16.a) Prove that \( \beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)} \).

b) Show that \( \frac{d}{dx} [x^n J_n(x)] = x^n J_{n-1}(x) \).

17.a) Find the inverse Laplace transform of \( \tan^{-1} \left( \frac{z}{s} \right) \).

b) Solve by the method of transforms, the equation \( y'' + 2y' - y' - 2y = 0 \), given \( y(0) = y'(0) = 0 \) and \( y''(0) = 6 \).