Code No. : **6003**

FACULTY OF ENGINEERING AND INFORMATICS

B.E. I Year (Common to all Branches) (Main) Examination, June 2010

MATHEMATICS – II

Time: 3 Hours] [Max. Marks: 75

*Note : Answer **all** questions of Part-A at **one** place in the Answer Book.*
*Answer **five** questions from Part-B.*

PART – A (Marks : **25**)

1. Solve : $xy' = (y-x)^3 + y$. **3**

2. Solve : $(x^3 + y^3 + 1)\,dx + xy^2 dy = 0$. **2**

3. Solve the differential equation $y^{iv} + 32y'' + 256y = 0$. **2**

4. Solve : $y'' - y' - 12y = 0, y(0) = 4, y'(0) = -5$. **3**

5. Find the Laplace transform of $f(t) = \sin h\ \omega t,\ t \geq 0$. **2**

6. Find the inverse Laplace transform of $\dfrac{s+3}{(s-1)(s+2)}$. **3**

7. Show that $P_n(1) = 1$. **2**

8. Define regular point and singular point of a differential equation. **3**

9. Evaluate : $\int_0^\infty x^2 e^{-x^2} dx$. **3**

10. Find the expression for $T_4'(x)$ in terms of $T_4(x)$ and $T_3(x)$. **2**

PART – B (5×10=50 Marks)

11. a) Solve $(3x^2y^3e^y + y^3 + y^2)\,dx + (x^3y^3e^y - xy)\,dy = 0$. 5

b) Find the orthogonal trajectories of the family of curves : $r = c(\sec\theta + \tan\theta)$. 5

12. a) Solve : $y''' - y'' + 4y' - 4y = 0,\ y(0) = 0,\ y'(0) = 3,\ y''(0) = -5$. 5

b) Solve $(D^2 - 2D + 1)\,y = x \sin hx$. 5

13. a) Apply convolution theorem to evaluate : 4

$$L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$$

b) Using Laplace transform, find the solution of :

$$y'' + 4y' + 4y = 12t^2e^{-2t},\ y(0) = 2,\ y'(0) = 1.$$ 6

14. Find the power series solution about the origin of the equation 10

$$(1-x^2)\,y'' - 4xy' + 2y = 0$$

15. a) Prove that $\dfrac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^{\infty} t^n P_n(t),\ t \neq 1$. 6

b) Show that $J_n'(x) = \frac{1}{2}\left[J_{n-1}(x) - J_{n+1}(x)\right]$. 4

16. a) Prove that $\Gamma\left(n+\dfrac{1}{2}\right) = \dfrac{\sqrt{\pi}\cdot\Gamma(2n+1)}{2^{2n}\,\Gamma(n+1)}$. 7

b) Prove that $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$. 3

17. a) Find the general solution of the equation $y'' + y = \operatorname{cosec} x$, using the method of variation of parameters. 5

b) State and prove the generating function of Chebyshev polynomials of the first kind. 5

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