Code No. 6003

FACULTY OF ENGINEERING & INFORMATICS

B.E.(Common to all Branches) I–Year (Supplementary) Examination, December 2013

Subject : Mathematics – II

Time : 3 hours **Max. Marks : 75**

Note: Answer all questions from Part-A. Answer any FIVE questions from Part-B.

PART – A (25 Marks)

1. Find an integrating factor of $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$. 2
2. Solve $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$ 3
3. Particular integral of $\frac{d^2y}{dx^2} + y = \cosh 3x$ is ________ 2
4. Solve $(D^2 + 4D + 5)y = 2e^{-2x}$. 3
5. Express 1 + x – x^2 in terms of Legendre's polynomials $P_n(x)$. 2
6. Show that $J_{1/2}(x) = J_{-1/2}(x) \tan x$. 3
7. Show that $U_{n+1}(x) = 2xU_n(x) - U_{n-1}(x)$. 2
8. Prove that $\Gamma n \, \Gamma(1-n) = \frac{\pi}{\sin(n\pi)}$ 3
9. Find the Laplace transform of $t^2 e^{-3t}$. 2
10. Apply convolution theorem to evaluate $L^{-1}\left\langle \frac{1}{s(s+1)} \right\rangle$ 3

PART – B (50 Marks)

11.a) Solve $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$. 5

b) Find the orthogonal trajectories of the family of curves

$\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$, λ being a parameter. 5

12.a) Solve $y'' + 4y' + 4y = 3\sin x + 4\cos x$, $y(0) = 1$ and $y'(0) = 0$. 5

b) Solve, by the method of variation of parameters. 5

$$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$$

13. Obtain the series solution of the equation.

$x(1-x)\frac{d^2y}{dx^2} - (1+3x)\frac{dy}{dx} - y = 0$ about $x = 0$ 10

14.a) Prove $\frac{1-z^2}{(1-2xz+z^2)^{3/2}} = \sum_{n=0}^{\infty}(2n+1)z^n P_n(x)$. 5

b) Show that $\int_{-1}^{+1} P_n^2(x) \, dx = \frac{2}{2n+1}$. 5

15.a) State and prove the orthogonal property of Chebyshev polynomials of the second kind. 5

b) Express $x^3 + 3x^2 - 2x + 1$ in terms of Chebyshev polynomials of the first kind. 5

16.a) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. 5

b) Show that $\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$. 5

17.a) Find the inverse Laplace transform of $\tan^{-1}\left(\frac{2}{s}\right)$ 4

b) Solve by the method of transforms, the equation 6

$y''' + 2y'' - y' - 2y = 0$, *given* $y(0) = y'(0) = 0$ and $y''(0) = 6$.

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