Code No. 6002 / M

FACULTY OF ENGINEERING and INFORMATICS

B.E. I – Year (Main) Examination, June 2014

Subject : Mathematics - I

Time : 3 hours **Max. Marks : 75**

Note: Answer all questions from Part-A. Answer any FIVE questions from Part-B.

PART – A (25 Marks)

1 Define rank of a matrix. (2)

2 Show that the vectors (1, 2, 3), (2, 3, 4) and (3, 4, 5) are linearly dependent. (3)

3 State the necessary condition for a positive series $\sum a_n$ to be convergent. (2)

4 Discuss the convergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$. (3)

5 Using the Lagrange mean value theorem, show that $|\sin b - \sin a| \le |b - a|$. (2)

6 Find the radius of curvature for the curve $y = x^2 - 6x + 10$ at (3, 1). (3)

7 Show that $\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^6 + y^2}$ does not exist. (2)

8 Expand $f(x, y) = x^2 + 3y^2 - 9x - 9y + 26$ in Taylor series of maximum order about (2, 2). (3)

9 Find ∇f, if $f(x, y, z) = \log_e (x^2 + y^2 + z^2)$ (2)

10 Show that the vector $(x^2 + yz)\,i + (4y - z^2x)j + (2xz - 4z)k$ is solenoidal (3)

PART – B (50 Marks)

11 a) Test for consistency and solve $2x - 3y + 7z = 5$, $3x + y - 3z = 13$, $2x + 19y - 47z = 32$. (5)

b) Verify Cayley - Hamilton theorem for the matrix $A = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$. (5)

12 a) Discuss the convergence of the series $\sum \left(\frac{n+2}{n+3}\right)^n x^n$ (5)

b) Test the series $\sum \left[\sqrt{n^4+1} - \sqrt{n^4-1}\right]$ for convergence. (5)

13 a) Verify Rolle's theorem for the function $f(x) = (x + 2)(x - 3)$ in the interval [-2, 3]. (5)

b) Find the evolute of the curve $x^2 = 4ay$. (5)

14 a) Find all asymptotes of the curve $y = x + \frac{1}{x}$. (5)

b) Discuss the maxima and minima of $f(x, y) = 4x^2 + 2y^2 + 4xy - 10x - 2y - 3$. (5)

15 a) Show that $\nabla^2 r^n = n(n+1)r^{n-2}$, where $r = |\bar{r}|$, $\bar{r} = xi + yj + zk$. (5)

b) If S is any closed surface enclosing a volume V and $\overline{F} = axi + byj + czk$, prove that $\int_S \overline{F} \cdot \overline{N}\, ds = (a + b + c)V$. (5)

16 a) Find the eigen values and the corresponding eigen vectors of $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 1 \\ 0 & 0 & 6 \end{pmatrix}$. (5)

b) Discuss convergence of $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} - \frac{1}{5^2} + \ldots\ldots$ (5)

17 Verify Green's theorem for $\int_C \left[(3x - 8y^2)dx + (4y - 6xy)dy\right]$, where C is the boundary of the region bounded by x = 0, y = 0 and x + y = 1. (10)

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