Code No. 6002

FACULTY OF ENGINEERING & INFORMATICS

B.E. I Year (Common to all Branches) (Suppl.) Examination, December 2013

Subject: Mathematics – I

Time: 3 Hours **Max.Marks: 75**

Note: Answer all questions from Part A. Answer any five questions from Part B.

PART – A (25 Marks)

1. Find the Taylor's series expansion of $f(x) = 2^x$ about x=0. (2)
2. Find the radius of curvature of the curve r = a sinθ + b cosθ at $\theta = \pi/2$. (3)
3. Show that $\lim_{(x,y)\to(0,0)} \frac{x^2 - x\sqrt{y}}{x^2 + y}$ does not exist. (2)
4. If z=y+f(u), $u = \frac{x}{y}$, show that $u\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$. (3)
5. Evaluate $\int_0^1 \int_{2y}^2 e^{x^2} dx\, dy$ by changing the order of integration. (2)
6. Find a vector that gives the direction of maximum rate of increase for f(x,y,z)=6xyz at (-1,2,1). (3)
7. Find the values of λ and μ such that the system of equations x+y+z = 6, x+2y+3z = 10, x + 2y + λz = μ has an infinite number of solutions. (2)
8. Show that the vectors (2,2,0), (3,0,2), (2,-2,2) are linearly independent. (3)
9. Discuss the convergence of the series $\sum (1+\frac{1}{n^p})^{n^{p+1}}$, p > 0. (2)
10. Test whether the series $\sum \frac{(-1)^n}{n\sqrt{n}}$ converges absolutely or not. (3)

PART – B (50 Marks)

11.(a) State and prove Rolle's theorem. (6)

(b) Find the envelope of the family of curves x tan α + y sec α = 5, α is a parameter. (4)

12.(a) Trace the curve $y = x^3 - 12x - 16$. (6)

(b) Examine $f(x,y) = x^4 + 2x^2y - x^2 + 3y^2$ for maximum and minimum values. (4)

13.(a) Show that $\bar{V} = 12xi - 15y^2 j + k$ is irrotational and find a scalar function f(x,y,z) such that $\bar{V}$ = grad f. (5)

(b) Use the divergence theorem to evaluate $\iint_S \bar{F}.\bar{n}\, ds$, where $\bar{F} = 4xi - 2y^2j + z^2k$ and S is the surface bounding the region $x^2+y^2 = 4$, z=0 and z=3. (5)

14.(a) If -4, 10, $\sqrt{2}$ are the three eigen values of A = $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 2 & 1 \\ 4 & 3 & 1 & 2 \end{pmatrix}$,

find the eigen values of A^{-1}. (4)

(b) Find the canonical form, nature, index and signature of the quadratic form

$Q = 8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 - 8x_2x_3 + 4x_3x_1$. (6)

15. Test the convergence of the series

a) $\frac{1}{1.3.5} + \frac{2}{3.5.7} + \frac{3}{5.7.9} + \ldots$ (4)

b) $\sum \frac{(n!)^2}{(2n)!} x^{2n}$ (6)

16.(a) Find the evolute of the curve $y^2=4ax$. (5)

(b) For the function f(x,y) = $\begin{cases} \frac{x^2y(x-y)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0 \quad, & (x,y) = (0,0), \end{cases}$

show that $\frac{\partial^2 f}{\partial x\, \partial y} \neq \frac{\partial^2 f}{\partial y\, \partial x}$ at (0,0). (5)

17.(a) Show that $\nabla \times (\nabla \times \bar{V}) = \nabla(\nabla \cdot \bar{V}) - \nabla^2 \bar{V}$. (5)

(b) Find the rank of the matrix A = $\begin{pmatrix} 2 & 3 & 1 & 0 & 4 \\ 3 & 1 & 2 & -1 & 1 \\ 4 & -1 & 3 & -2 & -2 \\ 5 & 4 & 3 & -1 & 5 \end{pmatrix}$. (5)

****